Introduction (From MA FW March 2011)

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include logarithmic, polynomial, rational, and radical functions in the Model Algebra II course. This course is comprised of standards selected from the high school **conceptual categories**, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course. Therefore, the complete standard is presented in the model course, with clarifying footnotes as needed to limit the scope of the standard and indicate what is appropriate for study in this particular course. Standards that were limited in Model Algebra I no longer have those restrictions in Model Algebra II. Students work closely with the expressions that define the functions, are facile with algebraic manipulations of expressions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

For the high school Model Algebra II course,[1] instructional time should focus on four critical areas: (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) expand understandings of functions and graphing to include trigonometric functions; (3) synthesize and generalize functions and extend understanding of exponential functions to logarithmic functions; and (4) relate data display and summary statistics to probability and explore a variety of data collection methods.

(1) A central theme of this Model Algebra II course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students explore the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Connections are made between multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The Fundamental Theorem of Algebra is examined.

(2) Building on their previous work with functions and on their work with trigonometric ratios and circles in the Model Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

(3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this Model Algebra II course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

(4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

[1] Adapted from the *Common Core State Standards for Mathematics* & *Appendix A: Designing High School Courses based on the Common Core State Standards for Mathematics*

The table below defines sequence of the Algebra I curriculum taught at Winthrop High School. The text below represents an overview of topics covered in this course. For more detailed information the reader should consult the CCSS - MA Frameworks (March 2011) document directly. Relevant content standards are included in the next section for your convenience.

Each quarter's durations should add up to 8 weeks, leaving 1 week of flexibility. The duration serves as a guide to the relative importance of each standard; however, quarterly boundaries are essential for common assessments.

	UNIT	THE STUDENT WILL…	DURATION (WEEKS)
QUARTER 1	***1) Review Algebra 1***	a) Solve equations & inequalities with various numeric representations. b) Model & solve real-life problems involving linear equations & functions from multiple perspectives: analytical, graphical, numerical & verbal.	2
	2) Linear Systems of Equations	a) Model & solve real-life problems involving a system of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Interpret the solution of a system of linear equations.	3
	3) Functions	a) Perform operations & model real life problems involving function notation & vocabulary from multiple perspectives: graphical, numerical, algebraic & verbal. b) Create, transform & invert various functions in multiple representations.	3
QUARTER 2	***4) Quadratic Equations & Functions***	a) Model & solve real-life problems involving quadratic equations. b) Interpret & create standard, vertex & factor forms of quadratic equations from multiple perspectives: graphical, numerical, algebraic & verbal. c) Solve quadratic functions using various form appropriate methods.	3
	5) Polynomials Functions	a) Utilize the vocabulary of polynomials as it pertains to graphical behaviors. b) Perform operations on polynomial: add, subtract, multiply, divide & factoring. c) Create & describe polynomial functions from graphs, factors & zeros.	3
	6) Powers, Roots & Radicals	a) Simplify & convert exponential & radical expressions using the rules of exponents, including rational exponents.	2
QUARTER 3	***7) Exponential Expressions & Functions***	a) Simplify & convert exponential & *logarithmic* expressions. b) Model & solve real-life problems involving exponential & *logarithmic* functions. c) Interpret & create exponential & *logarithmic* functions from multiple perspectives: graphical, numerical, algebraic & verbal. *Leave logarithmic expressions & functions until after the MCAS.*	3
	8) Probability	a) Model & solve real-life problems involving experimental & theoretical probability. b) Utilize counting methods & set theory to deepen probabilistic problem solving.	3
	9) Statistics	a) Create, utilize & interpret measures of central tendency & dispersion in various representations. b) Apply algebraic modeling to data that exhibits trends.	2
QUARTER 4	***10) MCAS Review***	a) Review various MCAS topics.	2
	11) Logarithmic Expressions &Functions	a) See "Exponential Expressions & Functions" above to relate to logarithmic content.	2
	12) Trigonometric Functions	a) Discover, explore & utilize the unit circle to right triangle trigonometry problems without a calculator. b) Utilize the unit circle to graph sine & cosine functions. c) Transform graphs of sine & cosine using y = A*TRIG(X).	2
	13) Rational Functions	a) Perform operations on rational expressions. b) Graph rational functions, including identification of vertical & horizontal asymptotes, from various representations: analytical, graphical, numerical & verbal.	2

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

The Standards for Mathematical Practice should be applied at every opportunity. The *italics* below are my interpretations of the SMP; however, please do read the actual text. The SMP themes are interconnected such that grouping is not only convenient but logical: (1 & 6), (2 & 3), (4 & 5), (7 & 8).

SMP-1 Make sense of problems and persevere in solving them

Allow students the time to try, make and fix mistakes.

SMP-2 Reason abstractly and quantitatively

Let the units of measure lead the way to solving the problem; make the connections between real world and symbols.

SMP-3 Construct viable arguments and critique the reasoning of others

Provide the opportunity for students to listen to their peer's explanations and argument critiques.

SMP-4 Model with mathematics

Provide opportunity for students to use prior knowledge when constructing a mathematical model.

SMP-5 Use appropriate tools strategically

Including but not limited to calculators; "It is a tool, not a crutch!"

SMP-6 Attend to precision

Let this be the over-arching theme: "Does the answer make sence?"

SMP-7 Look for and make use of structure

Allow students the time to recognize and make easier problems by breaking down difficult problems.

SMP-8 Look for and express regularity in repeated reasoning

Provide the opportunity for students to suffer through repeated operations until they discover the short-cut.

NOTATIONS

The + code indicates a standard beyond college and career readiness. These standards should be addressed in accelerated classes

Ro4* is a reference to a collection of standards (detailed at the end of this document) applied to Linear, Exponential and Quadratic functions.

Crossed out ~~content standards~~ are place-holders for future development.

DISCLAIMERS

Remember the transition to CCSS is a work in progress. Our student's success in obtaining the depth and breadth indicated in the following content standards depends on their previous perseverance and academic fortitude. At this time not all content standards may be completely achieved; however, it is our goal to deliver as much content as our students can absorb.

EDITORIAL COMMENTS

According to Dan Meyer the classroom is in need of a make-over. Watch his TED Talk video by searching for "Dan Meyer: Math class needs a makeover".

He recommends providing less information than a typical math textbook problem to gain student "buy-in" and encourage their critical thinking, starting with: "What information do we need to answer this problem?"

He also suggests that math teachers be "less helpful" to encourage students to gain strength in their own problem solving perseverance.

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 1	***1) Review Algebra 1***	a) Solve equations & inequalities with various numeric representations. b) Model & solve real-life problems involving linear equations & functions from multiple perspectives: analytical, graphical, numerical & verbal. *A-SSE-1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, ~~and simple rational~~ and exponential functions.* *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.*	2
	2) Linear Systems of Equations	a) Model & solve real-life problems involving a system of linear equations from multiple perspectives: graphical, numerical, algebraic & verbal. b) Interpret the solution of a system of linear equations. *A-CED-3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* *A-REI-6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Solve problems in real world context.* *A-REI-11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* ~~*N-VM-8+ Add, subtract, and multiply matrices of appropriate dimensions.*~~ ~~*N-VM-12+ Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.*~~ ~~*A-REI-8+ Represent a system of linear equations as a single matrix equation in a vector variable.*~~ ~~*A-REI-9+ Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).*~~	3
	3) Functions	a) Perform operations & model real life problems involving function notation & vocabulary from multiple perspectives: graphical, numerical, algebraic & verbal. b) Create, transform & invert various functions in multiple representations. *A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* *F-BF-1c+ Write a function that describes a relationship between two quantities. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. "* *F-BF-3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.* *F-BF-4 Find inverse functions. (a) Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x3 ~~or f(x) = (x + 1)/(x − 1) for x ≠ 1~~; (b)+ Verify by composition that one function is the inverse of another; (c)+ Read values of an inverse function from a graph or a table, given that the function has an inverse.(d)+ Produce an invertible function from a non-invertible function by restricting the domain.* *F-IF-7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (a) Graph linear and quadratic functions and show intercepts, maxima, and minima. (b) Graph piecewise-defined functions, including step functions and absolute value functions. (e) Graph exponential showing intercepts.*	3

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 2	***4) Quadratic Equations & Functions***	a) Model & solve real-life problems involving quadratic equations. b) Interpret & create standard, vertex & factor forms of quadratic equations from multiple perspectives: graphical, numerical, algebraic & verbal. c) Solve quadratic functions using various form appropriate methods. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *N-CN-1 Know there is a complex number i such that $i = \sqrt{-1}$, and every complex number has the form a + bi with a and b real.* *N-CN-7 Solve quadratic equations with real coefficients that have complex solutions.*	3
	5) Polynomials Functions	a) Utilize the vocabulary of polynomials as it pertains to graphical behaviors. b) Perform operations on polynomial: add, subtract, multiply, divide & factoring. c) Create & describe polynomial functions from graphs, factors & zeros. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *A-APR-3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.* *A-APR-2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).*	3
	6) Powers, Roots & Radicals	a) Simplify & convert exponential & radical expressions using the rules of exponents, including rational exponents. *N-RN-1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^(1/3) to be the cube root of 5 because we want (5^(1/3))^3=5^(1/3)3 to hold, so (5^(1/3))^3 must equal 5.* *N-RN-2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.*	2

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 3	***7) Exponential Expressions & Functions***	a) Simplify & convert exponential & *logarithmic* expressions. b) Model & solve real-life problems involving exponential & *logarithmic* functions. c) Interpret & create exponential & *logarithmic* functions from multiple perspectives: graphical, numerical, algebraic & verbal. *Leave logarithmic expressions & functions until after the MCAS.* *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *F-LE-4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*	3
	8) Probability	a) Model & solve real-life problems involving experimental & theoretical probability. b) Utilize counting methods & set theory to deepen probabilistic problem solving. *S-CP-1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").* ~~*S-CP-2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*~~ ~~*S-CP-3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.*~~ *S-IC-2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* *S-IC-3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*	3
	9) Statistics	a) Create, utilize & interpret measures of central tendency & dispersion in various representations. b) Apply algebraic modeling to data that exhibits trends. *S-IC-1 Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.* *S-IC-4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.* *S-IC-5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.* *S-IC-6 Evaluate reports based on data.* *S-ID-1 Represent data with plots on the real number line (dot plots, histograms, and box plots* *S-ID-2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.* ~~*S-ID-4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*~~	2

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
QUARTER 4	***10) MCAS Review***	a) Review various MCAS topics.	2
	11) Logarithmic Expressions &Functions	a) See "Exponential Expressions & Functions" above to relate to logarithmic content.	2
	12) Trigonometric Functions	a) Discover, explore & utilize the unit circle to right triangle trigonometry problems without a calculator. b) Utilize the unit circle to graph sine & cosine functions. c) Transform graphs of sine & cosine using y = A*TRIG(BX)+D. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *F-TF-1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.* *F-TF-2+ Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.* *F-TF-5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* ~~*F-TF-8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant.*~~	2
	13) Rational Functions	a) Perform operations on rational expressions. b) Graph rational functions, including identification of vertical & horizontal asymptotes, from various representations: analytical, graphical, numerical & verbal. *Ro4* A collection of standards (detailed at the end of this document) that links multiple representations (verbal, analytical, numerical & graphical) of functions and the required skills.* *A-REI-2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.* *A-APR-6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.* *A-APR-7+ Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.*	2

	UNIT	STUDENT OBJECTIVES & COMMON CORE CONTENT STANDARDS	DURATION (WEEKS)
*	*Rule of 4*	a) The following standards are applicable to the 3 major functions (linear, exponential and quadratic) of Algebra 1.The standards are grouped together here to minimize redundancy of separate listings in linear, exponential and quadratic units. *A-APR-1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.* *A-CED-1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from simple rational and exponential functions.* *A-CED-2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.* *A-REI-10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a cure (which could be a line).* *F-IF-4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *F-IF-5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *F-IF-6 Calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* *F-IF-7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (a) Graph linear and quadratic functions and show intercepts, maxima, and minima. (b) Graph piecewise-defined functions, including step functions and absolute value functions. (d) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. (e) Graph exponential & logarithmic functions, showing intercepts and end behaviors and trigonometric functions, showing period, midline and amplitude.* *F-IF-8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. (a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context. (MA.8.c Translate among different representations of functions and relations: graphs, equations, point sets, and tables.)* *F-IF-9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has a larger intercept.* *F-IF-MA-10 Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, or exponential.* *F-LE-1 Distinguish between situations that can be modeled with linear functions and with exponential functions. (a) Prove that linear functions grown by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.* *F-LE-2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).* *F-LE-3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.* *F-LE-5 Interpret the parameters in a linear function in terms of a context.*	Not all content standards apply to all instances of "Rule of 4"

Note: Rule of 4 Adopted from Algebra 1; may need revision to bring the depth and breadth up to Algebra 2 levels.